NANYANG TECHNOLOGICAL UNIVERSITY
NANYANG BUSINESS SCHOOL

BF2202 Mathematical Methods for Finance

Course Aims

This course is an introduction to mathematical finance. The course serves to equip students with basic mathematical knowledge and computational skills needed to solve problems related to finance. Furthermore, this course lays the foundation for more advanced topics, such as econometrics, stochastic calculus and optimal control theory, which make finance a very exciting subject.

Course Description

Modern finance draws upon many fields of mathematics, in particular, algebra, econometrics, numerical analysis, optimization theory, partial differential equations, probability theory, statistics and stochastic calculus. The diversity of mathematical skills needed to master finance makes it a very challenging subject for students.

The mathematical topics covered in this course include matrix algebra, calculus, difference equation, differential equation, optimization and portfolio mathematics (please refer to the proposed seminar schedule for details). The topics in the first five weeks of the course introduce

students to the simplest financial market model and explain arbitrage pricing in a discrete time framework. The topics in the next three weeks lay the foundations for continuous time finance. The topics in the last four weeks introduce students to optimization and portfolio theory. These selected topics serve as an introduction to some of the fields of mathematics that modern finance draws upon.

Course Learning Outcomes

By the end of this course, students will learn to use mathematical techniques to solve asset pricing and portfolio management problems. Specifically, students will be able to:

1. Construct and describe models for financial markets.
2. Analyse and identify arbitrage opportunities.
3. Model and price financial securities.
4. Explain and apply the Black-Scholes-Merton option pricing model.
5. Setup and perform optimization.
6. Construct and manage optimal portfolios.

Course Content

The key topics covered in this course are as follows:

1. Time Value of Money.
2. The Simplest Model of
3. Financial Markets.
4. Arbitrage and Pricing of Securities.
5. Univariate and Multivariate Calculus.
6. Black-Scholes-Merton Option Pricing Model.
7. Unconstrained and Constrained Optimization.
8. Portfolio Mathematics.
9. Eigenvalues and Eigenvectors

Course Assessments

Components	**Marks**	**Individual / Group**
C1: Assignments	10	Individual
C2: Participations	10	Individual / Group
C3: Quizzes	20	Individual
Final examination	60	Individual
Total	**100**	

Proposed Weekly Schedule

Week (Week beginning)	**Topics**	**Learning Outcomes**
1	Time Value of Money	1. Arithmetic & Geometric Series. 2. Simple & Compound Interest. 3. Limits & Exponential Function. 4. Continuous Compounding of Interest. 5. Present & Future Values. 6. Summation of Series. 7. Present Value of a Series of Cash Flows. 8. Annuity, Perpetuity & Amortization. 9. First Order Difference Equation. 10. Growth Models. 11. Solving Problems with Excel.
2	The Simplest Model of Financial Markets I	1. Time & State Modelling. 2. One-Period Finite State Model. 3. Securities & Their Pay-Offs. 4. Securities as Vectors. 5. Vector Space. 6. Operations on Securities. 7. Matrix as a Collection of Securities. 8. Transposition. 9. Matrix Multiplication and Portfolios. 10. Working with Matrices in Excel.

Week (Week beginning)	**Topics**	**Learning Outcomes**
3	The Simplest Model of Financial Markets II	1. Systems of Equations. 2. Hedging. 3. Linear Independence and Redundant Securities. 4. The Structure of the Marketed Subspace. 5. Identity Matrix and Arrow -Debreu Securities. 6. Matrix Determinant & Inverse. 7. Inverse Matrix and Replicating Portfolios. 8. Complete Market Hedging Formula. 9. Solving Linear Systems of Equations in Excel
4	Arbitrage and Pricing of Securities I	1. Hedging with Redundant Securities & Incomplete Market. 2. Asset Prices, Returns & Portfolio Units. 3. Arbitrage. 4. No-Arbitrage Pricing. 5. State Prices & the Arbitrage Theorem. 6. State Prices and Asset Returns. 7. Risk-Neutral Probabilities. 8. State Prices & No-Arbitrage Pricing.
5	Arbitrage and Pricing of Securities II	1. One-Period Binomial Model. 2. Multi-Period Binomial Model. 3. Computational Constraints. 4. Implementing the Binomial Model in Excel.

Week (Week beginning)	**Topics**	**Learning Outcomes**
6	Univariate Calculus **Quiz 1 covering topics in week 1–5**	1. Functions. 2. Payoff Functions of Derivatives. 3. Graphs. 4. Slope of a Function. 5. Continuity & Differentiability. 6. Differentiation. 7. Product Rule, Quotient Rule & Chain Rule. 8. Limits & L'Hospital Rule. 9. Sensitivity Analysis using Taylor & McClaurin Series. 10. Integration. 11. Integration by Substitutions & by Parts.
7	Multivariate Calculus	1. Partial Derivatives. 2. Total Differential. 3. Implicit Differentiation. 4. Fundamental Theorem of Calculus. 5. Working with Symbolic Math using Computer.
ζ	**Recess**	

Week (Week beginning)	**Topics**	**Learning Outcomes**
8	Black-Scholes-Merton Option Pricing Model	1. Differential Equation. 2. Partial Differential Equation. 3. Black-Scholes-Merton Option Pricing Model.
9	Unconstrained Optimization	1. Maxima & Minima of Univariate Functions. 2. First & Second Order Conditions for Optimality. 3. Convex & Concave Multivariate Functions. 4. Unconstrained Optima of Multivariate Functions. 5. Unconstrained Optimization using Excel.
10	Constrained Optimization	1. Optimization with Equality Constraints. 2. The Lagrangian Function. 3. The Meaning of the Lagrange Multiplier. 4. The Second Order Conditions for Optimality. 5. Optimization with Inequality Constraints. 6. Inequality Constraints & Karush-Kuhn-Tucker Conditions. 7. Constrained Optimization using Excel
11	Portfolio Mathematics I	1. Returns of Securities. 2. Expected Return & Variance-Covariance Matrix. 3. Mean-Variance Framework. 4. Portfolio Expected Return & Variance. 5. Quadratic Forms. 6. Basic Portfolio Problems 7. Minimum Variance Portfolio. 8. Minimizing Portfolio Risk for a Required Expected Return. Maximizing Portfolio Expected Return Given Tolerated Risk.
12	Portfolio Mathematics II **Quiz 2 covering topics in week 6–11**	9. Solving Portfolio Problems by Matrix Operations. 10. Solving Portfolio Problems by Row Operations. 11. Solving Portfolio Problems using Excel.

Week (Week beginning)	**Topics**	**Learning Outcomes**
13	Eigenvalues & Eigenvectors	1. Eigenvalues. 2. Eigenvectors. 3. Solving Linear Difference Equations. 4. Diagonalization of Matrices.